Mobile Communications

TCS 455

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Lecture 25

Office Hours:
BKD 3601-7
Tuesday 14:00-16:00
Thursday 9:30-11:30

Announcements

- Read the following from the SIIT online lecture note system
  - Section 1.2 from [Bahai, *Multi-carrier Digital Communications: Theory And Applications Of OFDM*, 2002]
  - Theory of PN Codes from [Karim and Sarraf, *W-CDMA and cdma2000 for 3G Mobile Networks*, 2002]
- Check the course web site for links to interesting resources about DFT.
- Communication Group Meeting: Presentation starts at 2:30 PM.
  - Move to the 4th floor quickly after lecture.

Chapter 5

OFDM

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Three steps towards modern OFDM

1. Mitigate Multipath (ISI) → Multicarrier modulation (FDM)
2. Gain Spectral Efficiency → Orthogonality of the carriers
3. Achieve Efficient Implementation → FFT and IFFT

- Completely eliminate ISI and ICI → Cyclic prefix

Chapter 5
OFDM

Cyclic Prefix (CP)

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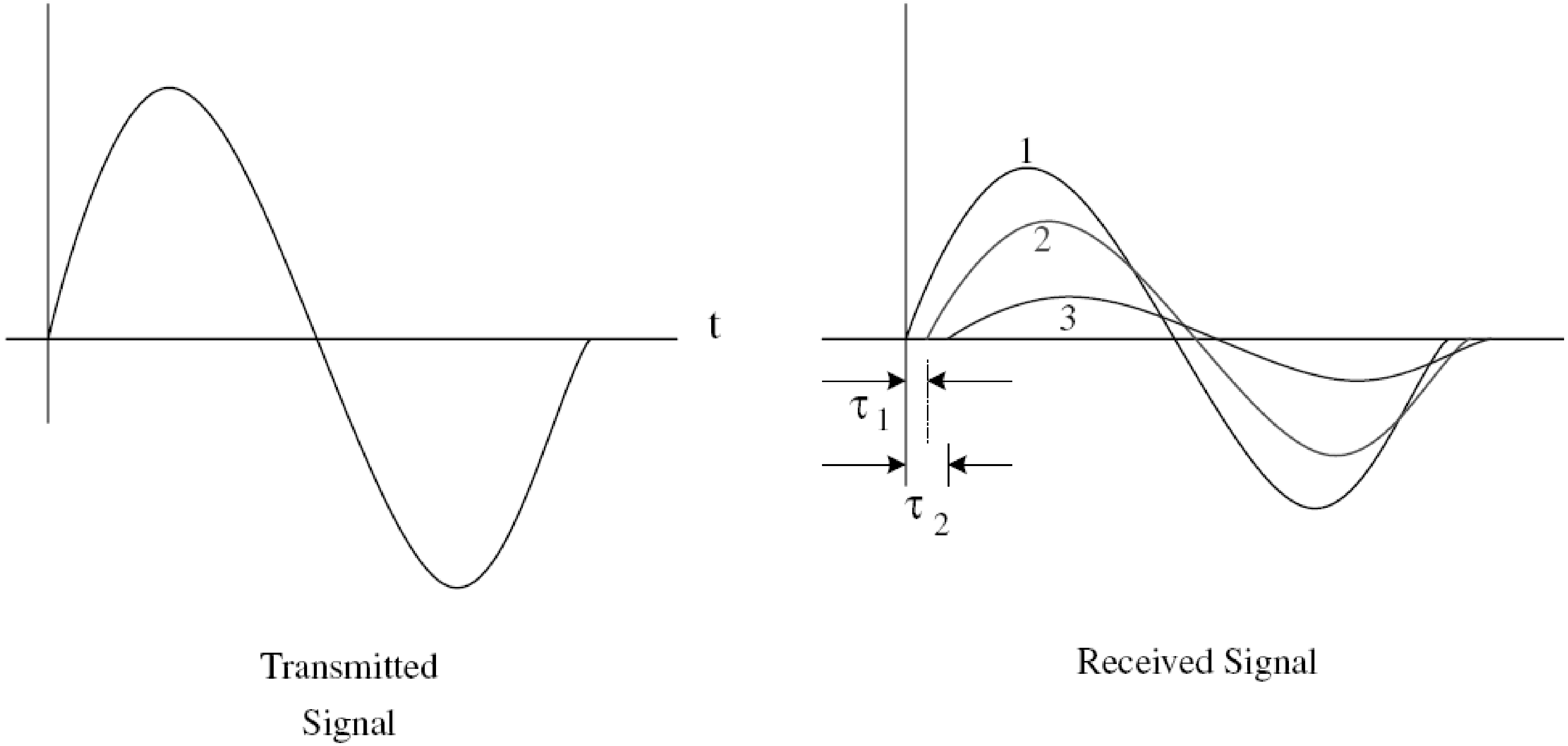
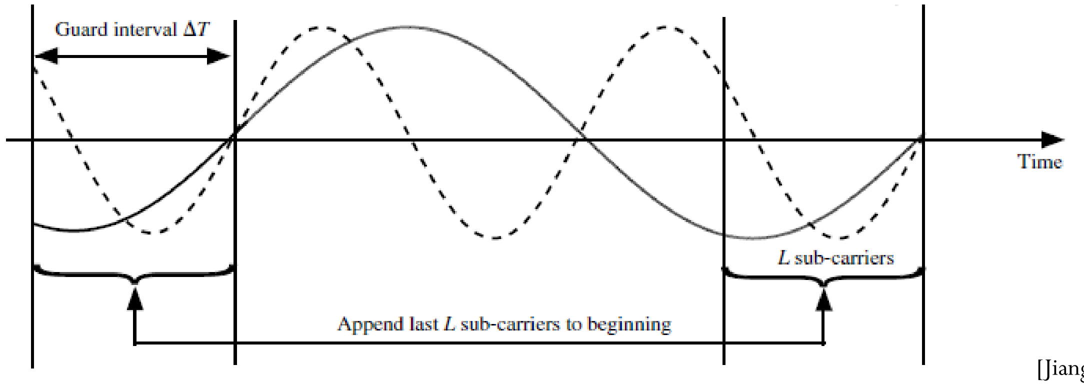
Cyclic Prefix: Motivation (1)



Cyclic Prefix: Motivation (2)

- When the number of sub-carriers increases, the OFDM symbol duration T_s becomes large compared to the duration of the impulse response τ_{max} of the channel, and the amount of ISI reduces.
- Can we "eliminate" the multipath (**ISI**) problem?
- To reduce the ISI, add **guard interval** larger than that of the estimated delay spread.
- If the guard interval is left empty, the orthogonality of the sub-carriers no longer holds, i.e., **ICI** (inter-channel interference) still exists.
- To prevent both the ISI as well as the ICI, OFDM symbol is **cyclically extended** into the guard interval.

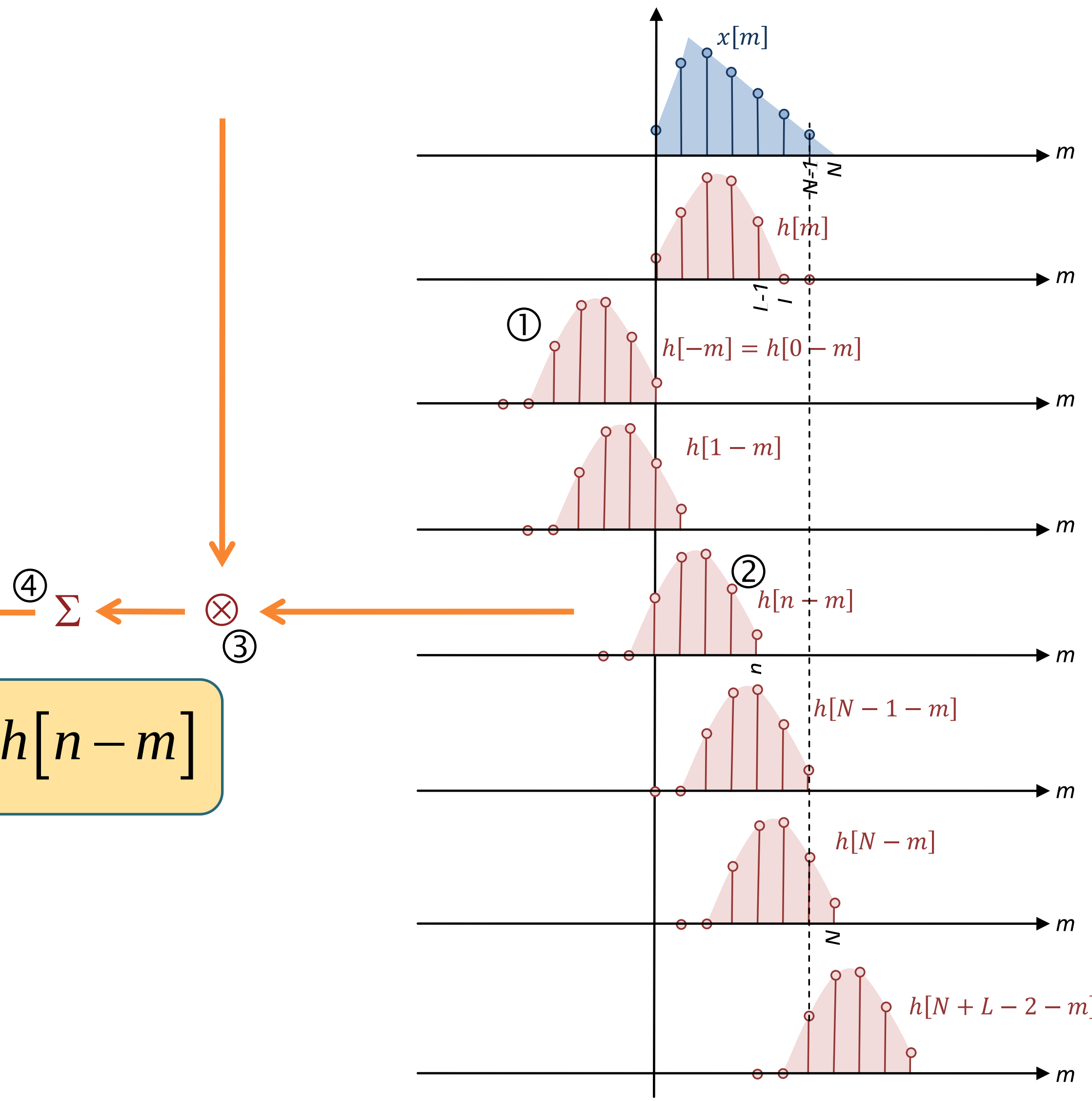


[Jiang]

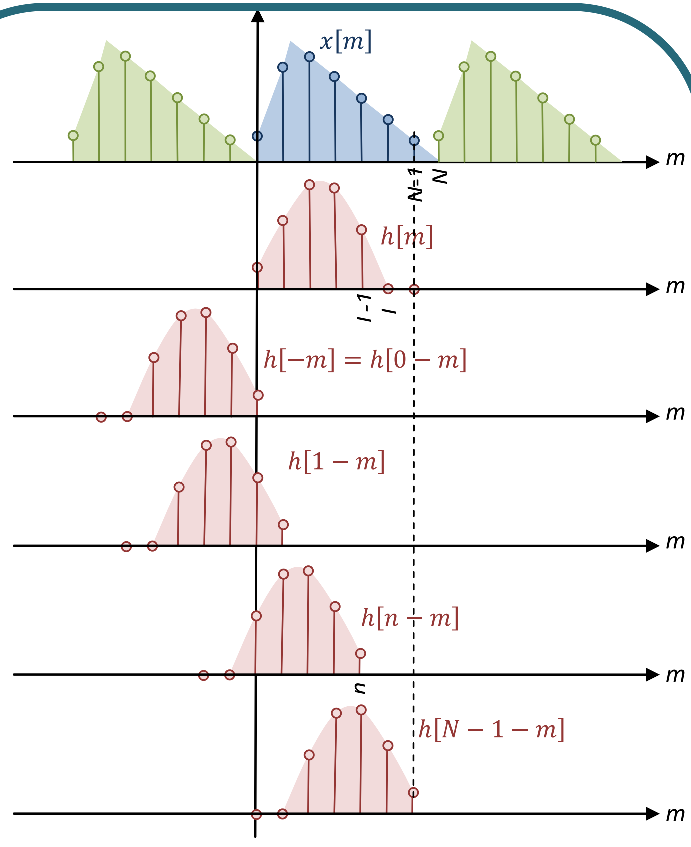
Convolution

① Flip
② Shift
③ Multiply
④ Add

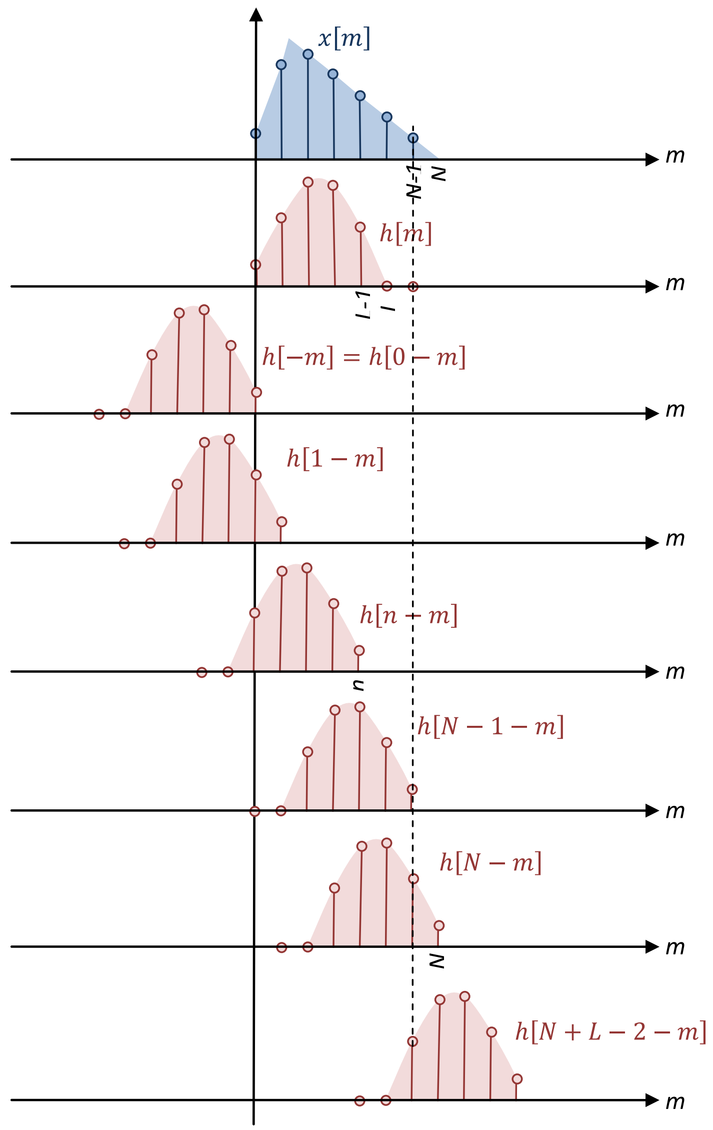
$$\{x * h\}[n] = \sum_m x[m]h[n-m]$$



Circular Convolution



Replicate *x* (now it looks periodic)
Then, perform the usual convolution
only on n = 0 to *N*-1



Circular Convolution: Example

Find

$$[1 \quad 2 \quad 3] * [4 \quad 5 \quad 6]$$

$$[1 \quad 2 \quad 3] \circledast [4 \quad 5 \quad 6]$$

$$[1 \quad 2 \quad 3 \quad 0 \quad 0] \circledast [4 \quad 5 \quad 6 \quad 0 \quad 0]$$

Discussion

- Circular convolution can be used to find the regular convolution by zero-padding.
- In modern OFDM, it is another way around.
- CTFT: convolution in time domain corresponds to multiplication in frequency domain.
- DFT: circular convolution in (discrete) time domain corresponds to multiplication in (discrete) frequency domain.
  - We want to have multiplication in frequency domain.
  - So, we want circular convolution and not the regular convolution.
- Real channel does regular convolution.
- With cyclic prefix, regular convolution can be used to create circular convolution.

Example

- Suppose $x^{(1)}$ = [1 -2 3 1 2] and h = [3 2 1]
- [1 -2 3 1 2] ⊛ [3 2 1 0 0] = [8 -2 6 7 11]
- [1 2 1 -2 3 1 2] * [3 2 1] = [3 8 8 -2 6 7 11 5 2]
- Suppose $x^{(2)}$ = [2 1 -3 -2 1]
- [2 1 -3 -2 1] ⊛ [3 2 1 0 0] = [6 8 -5 -11 -4]
- [-2 1 2 1 -3 -2 1] * [3 2 1] = [-6 -1 6 8 -5 -11 -4 0 1]
- [1 2 1 -2 3 1 2 -2 1 2 1 -3 -2 1] * [3 2 1] =

  [3 8 8 -2 6 7 11 5 2]

  \+

  [-6 -1 6 8 -5 -11 -4 0 1]

= [3 8 8 -2 6 7 11 -1 1 6 8 -5 -11 -4 0 1]

Circular Convolution: Key Properties

- Consider an N-point signal $x[n]$
- **Cyclic Prefix (CP) insertion**: If $x[n]$ is extended by copying the last ν samples of the symbols at the beginning of the symbol:

$$\hat{x}[n]=\begin{cases} x[n], & 0\le n\le N-1 \\ x[n+N], & -v\le n\le -1 \end{cases}$$

- Key Property 1:

$$\{h\circledast x\}[n]=(h*\hat{x})[n] \text{ for } 0\le n\le N-1$$

- Key Property 2:

$$\{h\circledast x\}[n]\xrightarrow{\text{FFT}} H_k X_k$$

OFDM with CP for Channel w/ Memory

- We want to send N samples $S_0, S_1, \ldots, S_{N-1}$ across noisy channel with memory.
- First apply IFFT: $S_k \xrightarrow{\text{IFFT}} s[n]$
- Then, add cyclic prefix

$$\hat{s} = \left[s[N-v], \ldots, s[N-1], s[0], \ldots, s[N-1] \right]$$

- This is inputted to the channel.
- The output is

$$y[n] = \left[p[N-v], \ldots, p[N-1], r[0], \ldots, r[N-1] \right]$$

- Remove cyclic prefix to get $r[n] = h[n] \circledast s[n] + w[n]$
- Then apply FFT: $r[n] \xrightarrow{\text{FFT}} R_k$
- By circular convolution property of DFT, $R_k = H_k S_k + W_k$

No ICI!

Example: 802.11a

Parameter	IEEE 802.11a
Bandwidth	20 MHz
Number of sub-carriers N_c	52 (48 data + 4 pilots) (64 FFT)
Symbol duration T_s	4 µs
Carrier spacing F_s	312.5 kHz $= \frac{1}{4-0.8\,[\mu s]}$
Guard time T_g	0.8 µs
Modulation	BPSK, QPSK, 16-QAM, and 64-QAM
FEC coding	Convolutional with code rate 1/2 up to 3/4
Max. data rate	54 Mbit/s